UNIVERSITY OF MANITOBA
Term Test 3C
COURSE: MATH 1500
DATE & TIME: November 26, 2018, 5:40PM – 6:40PM
CRN: various
DURATION: 1 hour EXAMINER: various

INSTRUCTIONS

I. No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

II. This exam has a title page, 4 pages of questions and two blank pages for rough work. Please check that you have all the pages. **DO NOT REMOVE THE SCRAP PAPER**

III. The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 50 points.

IV. **Answer all questions on the exam paper** in the space provided beneath the question. Unjustified answers will receive little or no credit. Only techniques taught in this course should be used. **Do not continue on the back of the page.** If you need more space, continue on one of the scrap pages, CLEARLY INDICATING THAT YOUR WORK IS TO BE CONTINUED.

V. Do not deface the QR - code in the top right corner. Doing so may result in the page not being scanned and therefore not graded.

Question	Points	Score
1	4	
2	2	
3	7	
4	6	
5	9	
6	22	
Total:	50	

UNIVERSITY OF MANITOBA
Term Test 3C
COURSE: MATH 1500
DATE & TIME: November 26, 2018, 5:40PM – 6:40PM
CRN: various
DURATION: 1 hour EXAMINER: various

---

[4] 1. Let $h(x) = \dfrac{g(x)+3}{2f(x)-2}$, Find $h'(2)$ if $f'(2) = g'(2) = -3$ and $f(2) = g(2) = 3$.

> **Solution:**
> Using the quotient rule for derivatives,
>
> $$\begin{aligned} h'(x) &= \frac{(g(x)+3)'(2f(x)-2) - (g(x)+3)(2f(x)-2)'}{(2f(x)-2)^2} \\ &= \frac{g'(x)(2f(x)-2) - (g(x)+3)(2f'(x))}{(2f(x)-2)^2} \end{aligned}$$
>
> Therefore,
>
> $$\begin{aligned} h'(2) &= \frac{g'(2)(2f(2)-2) - (g(2)+3)(2f'(2))}{(2f(2)-2)^2} \\ &= \frac{(-3)(2(3)-2) - (3+3)(2(-3))}{(2(3)-2)^2} = \frac{3}{2} \end{aligned}$$

[2] 2. Given that, for a function $f(x)$, $f''(x)$ is continuous near $x = c$, $f'(c) = 0$, and $f''(c) > 0$, what can we conclude about $f(c)$ in terms of relative (local) extrema? Explain your answer.

> **Solution:** Since $f'(c) = 0$, f is defined at $x = c$ and has a critical number at $x = c$. Since $f''(c) > 0$, according to the second derivative test, f has a relative (local) minimum at $x = c$.

UNIVERSITY OF MANITOBA
Term Test 3C
COURSE: MATH 1500
DATE & TIME: November 26, 2018, 5:40PM – 6:40PM
CRN: various
DURATION: 1 hour EXAMINER: various

---

[7] 3. Find the absolute extrema of $f(x) = -4x^3 + x^4$ on the interval $[-1, 4]$. Justify your answer.

Solution: Since f is a polynomial function, it is continuous on the closed interval $[-1, 4]$.

$$f'(x) = -12x^2 + 4x^3$$

which is defined everywhere.
To find critical numbers, we have to solve $f'(x) = 0$.

$$f'(x) = 0 \Rightarrow -12x^2 + 4x^3 = 0 \Rightarrow -4x^2(3 - x) = 0 \Rightarrow x = 0 \text{ or } x = 3.$$

Hence, with the endpoints, we test $x = 0$ and $x = 3$, since they are both in the interval.

$$\begin{aligned} f(-1) &= 5 \\ f(0) &= 0 \\ f(3) &= -27 \\ f(4) &= 0. \end{aligned}$$

Then, the absolute maximum is 5 and the absolute minimum is -27.

[6] 4. Suppose that $f(x)$ is differentiable on an interval I and $f'(x) = 0$. Show that $f(x)$ is constant on I.

Solution: Let x_1 and x_2 be any two distincts numbers in I such that $x_1 < x_2$. Since $f'(x)$ exists on I, $f(x)$ is differentiable on I and thus it is continuous on I. Therefore, we have that $f(x)$ is continuous on $[x_1, x_2]$ and differentiable on (x_1, x_2).
So by the Mean Value Theorem, there exists c between x_1 and x_2 such that

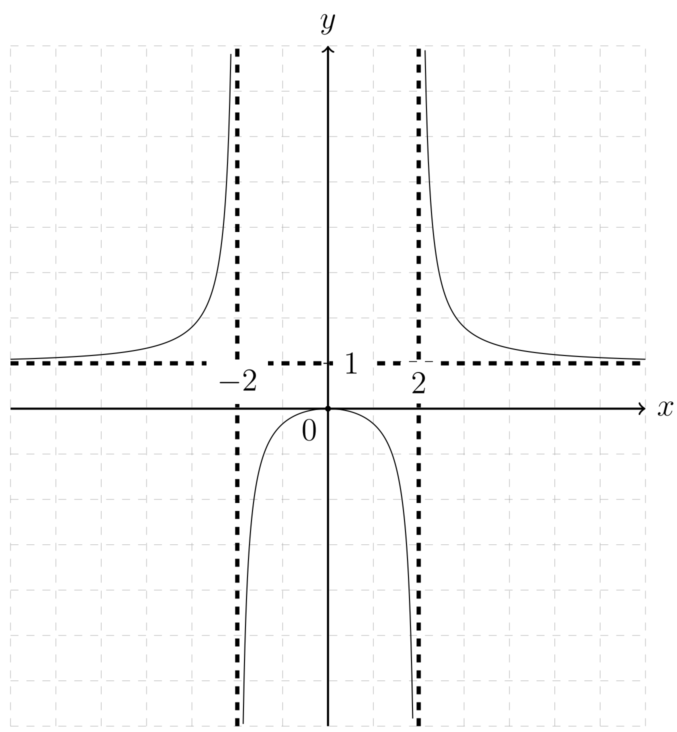
$$f(x_2) - f(x_1) = f'(c)(x_2 - x_1).$$

Since $f'(c) = 0$, then $f(x_2) - f(x_1) = 0$ and, hence, $f(x_2) = f(x_1)$. Thus, $f(x)$ is constant on I.

UNIVERSITY OF MANITOBA
Term Test 3C
COURSE: MATH 1500
DATE & TIME: November 26, 2018, 5:40PM – 6:40PM
CRN: various
DURATION: 1 hour EXAMINER: various

---

5. The graph of the function $f(x) = \dfrac{-x^2}{4 - x^2}$ is given below. Use this graph to gather information and fill in the blanks. (*If a feature doesn't apply, write "None."*)



[1] (a) Equation(s) of any vertical asymptotes: $\underline{x = 2, x = -2}$

[1] (b) Equation(s) of any horizontal asymptotes: $\underline{y = 1}$

[1] (c) Open intervals where f is increasing: $\underline{(-\infty, -2),\ (-2, 0)}$

[1] (d) Open intervals where f is decreasing: $\underline{(0, 2),\ (2, \infty)}$

[1] (e) x and y-coordinates of any local maxima: $\underline{(0, 0)}$

[1] (f) x and y-coordinates of any local minima: <u>None</u>

[1] (g) Open intervals where f is concave up: $\underline{(-\infty, -2),\ (2, \infty)}$

[1] (h) Open intervals where f is concave down: $\underline{(-2, 2)}$

[1] (i) x and y-coordinates of any inflection point(s): <u>None</u>

UNIVERSITY OF MANITOBA
Term Test 3C
COURSE: MATH 1500
DATE & TIME: November 26, 2018, 5:40PM – 6:40PM
CRN: various
DURATION: 1 hour EXAMINER: various

---

6. Use the function $f(x)$, the first derivative $f'(x)$ and the second derivative $f''(x)$ as defined here to gather information and fill in the blanks below. (*If a feature doesn't apply, write "None."*)

$$f(x) = \frac{x^2 - 2x + 2}{x - 1} \qquad f'(x) = \frac{x(x-2)}{(x-1)^2} \qquad f''(x) = \frac{2}{(x-1)^3}$$

[1] (a) Domain of f: $\underline{(-\infty, 1) \cup (1, \infty)}$

[1] (b) Symmetry of f: <u>None</u>

[1] (c) x-intercepts: <u>None</u>

[1] (d) y-intercept: $\underline{-2}$

[1] (e) Equation(s) of any vertical asymptotes: $\underline{x = 1}$

[1] (f) Equation(s) of any horizontal asymptotes: <u>None</u>

[1] (g) x and y-coordinates of any critical point(s): $\underline{(0, -2), (2, 2)}$

[2] (h) Open intervals where f is increasing: $\underline{(-\infty, 0),\ (2, \infty)}$

[2] (i) Open intervals where f is decreasing: $\underline{(0, 1), (1, 2)}$

[1] (j) x and y-coordinates of any local maxima: $\underline{(0, -2)}$

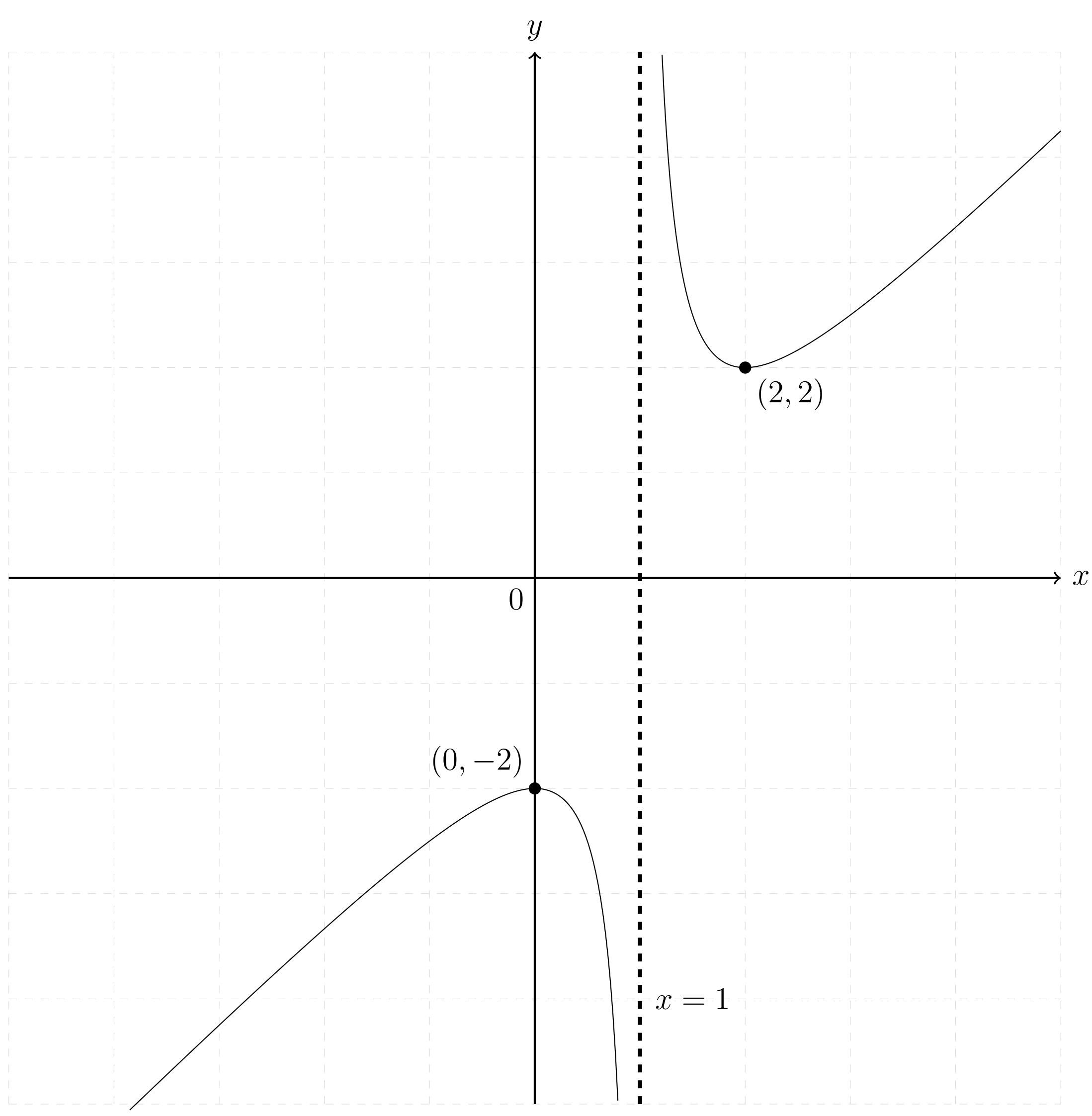
[1] (k) x and y-coordinates of any local minima: $\underline{(2, 2)}$

[2] (l) Open intervals where f is concave up: $\underline{(1, \infty)}$

[2] (m) Open intervals where f is concave down: $\underline{(-\infty, 1)}$

[1] (n) x and y-coordinates of any inflection point(s): <u>None</u>

[4] (o) Use the information from the previous parts to give a neat sketch of the graph $y = f(x)$, making sure that you label all important features of the graph.



[Scrap page]

If you are using this page to continue your work from a previous question, clearly indicate on the original page that your work is continuing here. Otherwise, your work will not be marked.

UNIVERSITY OF MANITOBA
Term Test 3C
COURSE: MATH 1500
DATE & TIME: November 26, 2018, 5:40PM – 6:40PM
CRN: various
DURATION: 1 hour EXAMINER: various

[Scrap page]

If you are using this page to continue your work from a previous question, clearly indicate on the original page that your work is continuing here. Otherwise, your work will not be marked.